A SIMP based Methodology for Topology Optimization and its Application to Piezoelectric Energy Harvester Designs

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The solid isotropic material with penalization (SIMP) method has been widely employed in topology optimizations for its effectiveness and low computational cost. However, its final optimized topology that contains a quite lot of gray elements is often blurry, which is not feasible and cannot be embraced by a designer. In this paper, a topology optimization methodology based on SIMP and radial basis function (RBF) is proposed. The SIMP is executed to obtain a preliminary optimized topology, and a post-processor based on RBF is then used to smooth the topology of the blurry area of the preliminary topology. To validate the proposed method, it is applied to the topology optimization of a piezoelectric energy harvester. The numerical results have demonstrated that the proposed methodology can improve the performance of the SIMP method and yield more feasible topologies.

Index Terms—**Coupled problems, multi-physics, SIMP method, topology optimization.**

I. Proposed Methodology for Topology Optimization

Topology optimization (TO) is a very promising technique for a product design since an initial design can be obtained beforehand. In this regard, the researchers in computational electromagnetics have devoted a wealth of efforts to develop topology optimization techniques, such as the density-based, the ON/OFF, the level-set, and the evolutionary optimization methods, for electromagnetic device designs.

A micro-electromechanical system (MEMS) based on its miniaturization and good suitability has attracted significant attentions in remote and embedded system applications. As one of the MEMSs to convert mechanical vibrational energy to electrical energy, a piezoelectric energy harvesting (PEH) device shows great potential in remote sensors and embedded devices due to its compact size [1]-[2].

In this paper, a SIMP method is proposed for the topology optimization of a typical unimorph piezoelectric energy harvester. The effect of parameters in material interpolation model is analyzed. Besides, as a density-based method, the optimized topology of a SIMP method is often blurry, since many gray elements are included. To solve this issue, a post-processor based on RBF is proposed. The optimized topology of a PEH under static and harmonic loads is obtained.

A. Performance Computation

For the performance computation, a 3D finite element model of piezoelectric energy harvester is proposed. Electrodes are deposited on the piezoelectric parts and imposed equipotential electrical conditions. Influence of the electrodes on the mechanic performance is negligible. The coupled mechanical and electric field formulation is given as

$$\begin{aligned}&\mathbf{M}_{uu}\ddot{\mathbf{U}}+\mathbf{K}_{uu}\mathbf{U}+\mathbf{K}_{u\varphi}\mathbf{\Phi}=\mathbf{F}\\&\mathbf{K}_{\varphi u}\mathbf{U}-\mathbf{K}_{\varphi\varphi}\mathbf{\Phi}=\mathbf{Q}\end{aligned}\tag{1}$$

where $\mathbf{K}_{u\varphi}=(\mathbf{K}_{\varphi u})^{\mathrm{T}}$ represents the piezoelectric coupling matrix, $\mathbf{K}_{uu}$ and $\mathbf{K}_{\varphi\varphi}$ denote the structural stiffness and dielectric conductivity matrices, $\mathbf{M}_{uu}$ is the structural mass matrix, $\ddot{\mathbf{U}}$ denotes the displacement acceleration vectors, $\mathbf{U}$ and $\mathbf{\Phi}$ denote the displacement and electric potential vectors, $\mathbf{F}$ and $\mathbf{Q}$ are the applied force and electric charge vectors.

Since the applied electric charge is zero, $\mathbf{Q}$ is always zero. To consider the structural damping, Rayleigh damping is added to the matrix. Electric circuits are described by using the equivalent capacitance matrix method and the resultant matrices are compromised into the dielectric matrix. Equation (1) can thus be formulated as

$$\begin{aligned}&\mathbf{M}_{uu}\ddot{\mathbf{U}}+\mathbf{C}_{uu}\dot{\mathbf{U}}+\mathbf{K}_{uu}\mathbf{U}+\mathbf{K}_{u\varphi}\mathbf{\Phi}=\mathbf{F}\\&\mathbf{K}_{\varphi u}\mathbf{U}-(\mathbf{K}_{\varphi\varphi}+\hat{\mathbf{K}}_{\varphi\varphi})\mathbf{\Phi}=0\end{aligned}\tag{2}$$

where $\mathbf{C}_{uu}$ is the damping matrix, $\hat{\mathbf{K}}_{\varphi\varphi}$ is the effective capacitance matrix.

B. A SIMP Method

In the SIMP method, a power-law interpolation function is introduced to relax a large-scale discrete variable TO problem to a continuous variable one [3], and then identifies the optimization means iteratively, leading the continuous design towards a solid/void solution. To apply the SIMP method to piezoelectric material, the following interpolation model [2] is adopted

$$\begin{aligned}&C=\gamma^{p_1}C_0\\&e=\gamma^{p_2}e_0\\&\varepsilon=\gamma^{p_3}\varepsilon_0\\&\rho=\gamma\rho_0\end{aligned}\tag{3}$$

where γ ($0<\gamma\le 1$) is the pseudo-density as a design variable which differs in each finite element; C_0, e_0, and ε_0 are the elasticity, piezoelectric, and permittivity matrices of the piezoelectric material with penalization factors p_1, p_2 and p_3, respectively; ρ_0 is the density of the piezoelectric material. The effect of different combinations of the penalization factors on the optimized topology obtained will be analyzed in full paper for space limitations.

C. RBF Post-Processor

To smooth the boundary of the optimized topology to make it more feasible for engineering practice, a post-processor based on RBF is proposed. Given a series of sampling points and the corresponding function values, (x_j, f_j), and the radial basis function H, which is a monotonic function regarding to a n-dimensional Euclidean space $||\bullet||$, the interpolation of function $f(x)$ can be formulated as [4]

$$f(x) = \sum_{j=1}^{N} c_j H(|| x - x_j ||) \quad (4)$$

where $|| x - x_i ||$ is the Euclidean norm between point x and center x_j, c_j is the j^{th} coefficient. In this paper, the multiquadrics function is adopted as the RBF function, namely

$$H(|| x - x_i ||) = \sqrt{(|| x - x_i ||)^2 + h} \quad (5)$$

where h is a control parameter. After a series of sampling points are input as the trained samples, the coefficient c_j (j=1,2,...,N) is obtained, and the final solution at a given point x is reconstructed using (4).

D. Iterative Procedures of Proposed Methodology

In the proposed topology optimization methodology, the SIMP method is first adopted to obtain an initial optimal topology. 3D finite element and sensitivity analysis are developed, and the optimality criteria (OC) method is employed in the SIMP. The RBF post-processor is executed to smooth the blurry topology. The iterative procedures of the proposed methodology are shown in Fig .1.

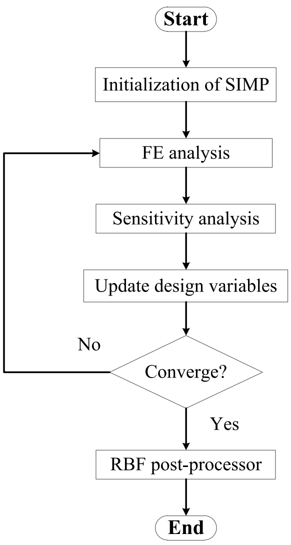

Fig. 1. Flow chart of optimization procedure

II. Optimization of a Unimorph Energy Harvester

The schematic diagram of a unimorph energy harvester is given in Fig .2. The energy conversion factor is chosen as the optimal objective to be maximized.

Fig .3 shows the blurry optimized topology obtained by using the SIMP under a static analysis case, Fig. 4 and Fig. 5 denote the pixel hard boundary and the smooth boundary after the post-processing respectively. For the easiness of demonstrations, only the vertical views are presented here. The energy conversion factor of the initial topology is 2.50%, of the optimized topology with a hard boundary 3.23% and a smooth topology after RBF 3.51%; Besides, 40% material cost of the initial topology design is reduced.

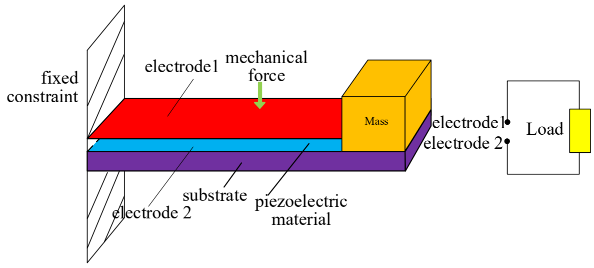

Fig. 2. The schematic diagram of a unimorph energy harvester

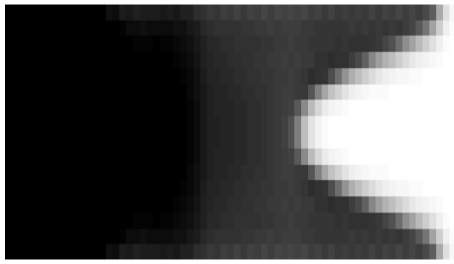

Fig. 3. Optimized topology of piezoelectric layer by SIMP

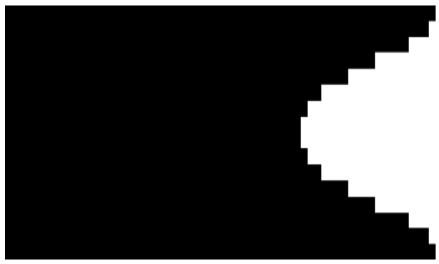

Fig. 4. Optimized topology with pixel hard boundary

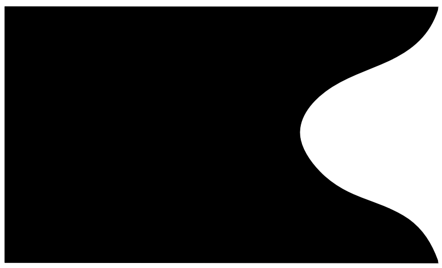

Fig. 5. Optimized topology using RBF post-processing

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